

# Bits, Bytes, and Integers

15-213: Introduction to Computer Systems  
2<sup>nd</sup> and 3<sup>rd</sup> Lectures, Sep 1 and Sep 6, 2011

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# Encoding Byte Values

## ■ Byte = 8 bits

- Binary  $00000000_2$  to  $11111111_2$
- Decimal:  $0_{10}$  to  $255_{10}$
- Hexadecimal  $00_{16}$  to  $FF_{16}$ 
  - Base 16 number representation
  - Use characters '0' to '9' and 'A' to 'F'
  - Write  $FA1D37B_{16}$  in C as
    - `0xFA1D37B`
    - `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

# Encoding Example (Cont.)

$x =$             15213: 00111011 01101101  
 $y =$             -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
<b>Sum</b>	<b>15213</b>		<b>-15213</b>	

# Values for Different Word Sizes

	W			
	8	16	32	64
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

## ■ Observations

- $|TMin| = TMax + 1$ 
  - Asymmetric range

# Two's Complement Addition

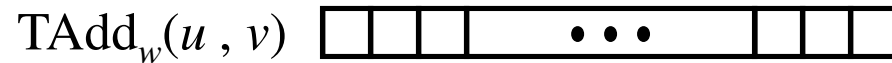
Operands:  $w$  bits



True Sum:  $w+1$  bits



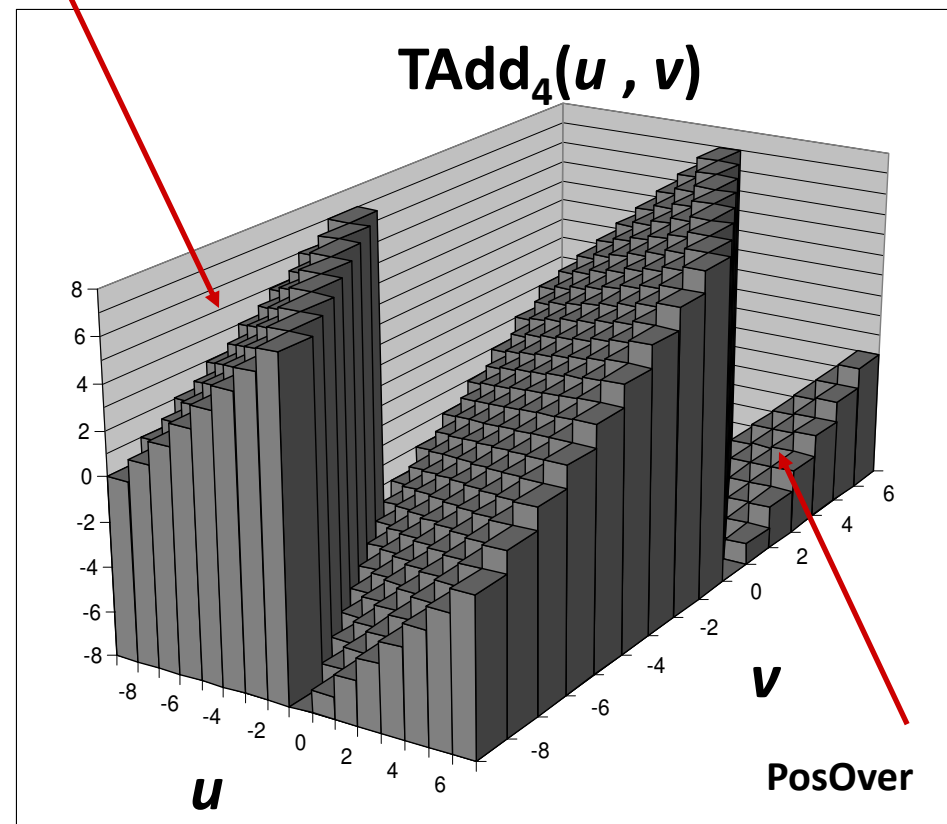
Discard Carry:  $w$  bits



# Visualizing 2's Complement Addition

- **Values**
  - 4-bit two's comp.
  - Range from -8 to +7
- **Wraps Around**
  - If  $\text{sum} \geq 2^{w-1}$ 
    - Becomes negative
    - At most once
  - If  $\text{sum} < -2^{w-1}$ 
    - Becomes positive
    - At most once

NegOver



# Negation: Complement & Increment

- Claim: Following Holds for 2's Complement

$$\sim x + 1 == -x$$

- Complement

- Observation:  $\sim x + x == 1111\dots111 == -1$

$$\begin{array}{r}
 x \quad \boxed{10011101} \\
 + \quad \sim x \quad \boxed{01100010} \\
 \hline
 -1 \quad \boxed{11111111}
 \end{array}$$

- Complete Proof?