

Bits, Bytes, and Integers

15-213: Introduction to Computer Systems
2nd and 3rd Lectures, Sep 1 and Sep 6, 2011

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Encoding Byte Values

■ Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
- Hexadecimal 00_{16} to FF_{16}
 - Base 16 number representation
 - Use characters '0' to '9' and 'A' to 'F'
 - Write $FA1D37B_{16}$ in C as
 - `0xFA1D37B`
 - `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Boolean Algebra

■ Developed by George Boole in 19th Century

- Algebraic representation of logic
 - Encode “True” as 1 and “False” as 0

And

- $A \& B = 1$ when both $A=1$ and $B=1$

$\&$	0	1
0	0	0
1	0	1

Or

- $A | B = 1$ when either $A=1$ or $B=1$

	0	1
0	0	1
1	1	1

Not

- $\sim A = 1$ when $A=0$

\sim	
0	1
1	0

Exclusive-Or (Xor)

- $A \wedge B = 1$ when either $A=1$ or $B=1$, but not both

\wedge	0	1
0	0	1
1	1	0

General Boolean Algebras

■ Operate on Bit Vectors

- Operations applied bitwise

01101001	01101001	01101001	01101001
& 01010101	01010101	^ 01010101	~ 01010101
01000001	01111101	00111100	10101010

■ All of the Properties of Boolean Algebra Apply

Shift Operations

■ Left Shift: $x \ll y$

- Shift bit-vector x left y positions
 - Throw away extra bits on left
 - Fill with 0's on right

■ Right Shift: $x \gg y$

- Shift bit-vector x right y positions
 - Throw away extra bits on right
- Logical shift
 - Fill with 0's on left
- Arithmetic shift
 - Replicate most significant bit on left

Argument x	01100010
$\ll 3$	00010000
Log. $\gg 2$	00011000
Arith. $\gg 2$	00011000

Argument x	10100010
$\ll 3$	00010000
Log. $\gg 2$	00101000
Arith. $\gg 2$	11101000

Encoding Example (Cont.)

$x =$ 15213: 00111011 01101101
 $y =$ -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		15213		-15213

Values for Different Word Sizes

	W			
	8	16	32	64
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

■ Observations

- $|TMin| = TMax + 1$
 - Asymmetric range

Two's Complement Addition

Operands: w bits



True Sum: $w+1$ bits



Discard Carry: w bits



Visualizing 2's Complement Addition

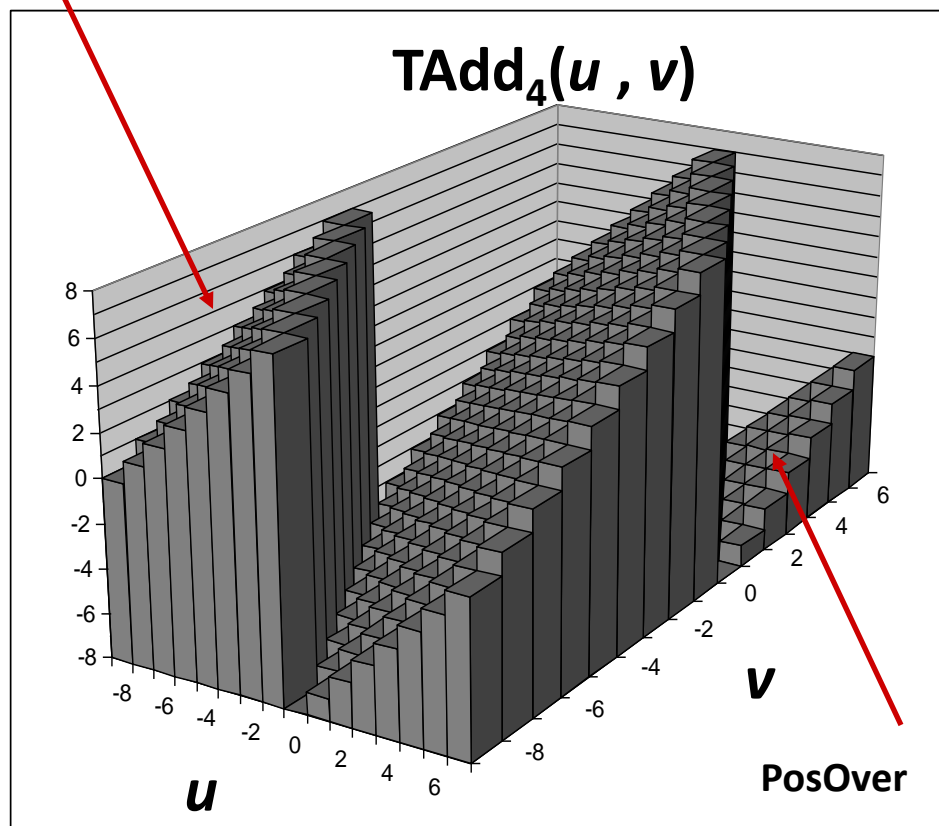
■ Values

- 4-bit two's comp.
- Range from -8 to +7

■ Wraps Around

- If $\text{sum} \geq 2^{w-1}$
 - Becomes negative
 - At most once
- If $\text{sum} < -2^{w-1}$
 - Becomes positive
 - At most once

NegOver



Negation: Complement & Increment

- Claim: Following Holds for 2's Complement

$$\sim x + 1 == -x$$

- Complement

- Observation: $\sim x + x == 1111\dots111 == -1$

$$\begin{array}{r}
 x \quad 10011101 \\
 + \quad \sim x \quad 01100010 \\
 \hline
 -1 \quad 11111111
 \end{array}$$

- Complete Proof?